A new measurement of K^+_{e4} decay and the s-wave $\pi\pi$-scattering length a_0^0

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A sample of $4 \cdot 10^5$ events from the decay $K^+ \to \pi^+\pi^- e^+\nu_e$ (K_{e4}) has been collected in experiment E865 at the Brookhaven AGS. The analysis of these data yields new measurements of the K_{e4} branching ratio ($(4.11 \pm 0.01 \pm 0.11) \cdot 10^{-5}$), the s-wave $\pi\pi$ scattering length ($a_0^0 = 0.228 \pm 0.012 \pm 0.003$), and the form factors F, G, and H of the hadronic current and their dependence on the invariant $\pi\pi$ mass.

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More than thirty years ago it was recognized that measurements of the properties of K_{e4} decay ($K^\pm \to \pi^+\pi^- e^\pm \nu_e(\bar{\nu}_e)$) would provide important information about both the weak and strong interactions. This four-body semileptonic decay is particularly interesting because the two pions are the only hadrons in the final state. It allows studies over a broad kinematic range of several form factors describing both the vector and axial vector hadronic currents, and uniquely of the low energy $\pi\pi$ interaction in an environment without presence of other hadrons.

While experimental studies of K_{e4} held promise of significant physics insight, the small branching ratio of about 0.004% has made precise measurements of the decay parameters difficult [1,2]. For instance, while the possibility of extracting the isospin zero, angular momentum zero scattering length a_0^0 has long been recognized [3], it was not until 1977, when the Geneva-Saclay experiment [2] gathered about 30,000 events, that a measurement was made of this quantity to 20% accuracy.

On the theoretical side, chiral QCD perturbation theory (ChPT) [4] makes firm predictions for the scattering length. The tree level calculation in ChPT using current algebra techniques in the soft-pion limit yields $a_0^0 = 0.156$ (in units of m_π) [5]. The one-loop ($a_0^0 = 0.201 \pm 0.01$ [6]) and two loop calculations ($a_0^0 = 0.217$ [7]) show a satisfactory convergence. The most recent calculation [8] matches the known chiral perturbation theory representation of the $\pi\pi$ scattering amplitude to two loops [7] with a phenomenological description that relies on the Roy equations [9,10], resulting in the prediction $a_0^0 = 0.220 \pm 0.005$.

The analysis of the Geneva-Saclay experiment [2] combined with the Roy equations and the inclusion of peripheral $\pi N \to \pi\pi N$ data led to the presently accepted value of $a_0^0 = 0.26 \pm 0.05$ [11]. It has been argued, that, if the central experimental value $a_0^0 = 0.26$ would be confirmed with a smaller error, such a large value can only be explained by a significant reduction of the quark condensate $< 0|\bar{u}u|0 >$, as is possible in generalized chiral perturbation theory (GChPT) [12]. The quark condensate is an order parameter measuring the spontaneous breaking of chiral symmetry. Vice versa measuring a_0^0 with higher precision allows to reduce the bounds on this parameter [13].

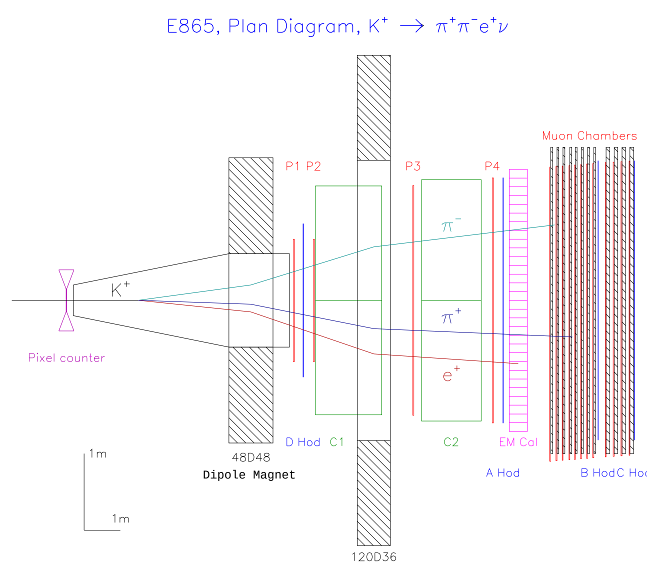

FIG. 1. Plan view of the E865 detector. A K_{e4} event is superimposed.

The analysis outlined here is based on data recorded at

the Brookhaven AGS, employing the E865 detector. The apparatus, described in detail in [14], is shown in Figure 1. The detector resided in a 6 GeV/c unseparated K^+ beam directly downstream of a 5 m long evacuated decay volume. A first dipole magnet separated the K^+ decay products by charge. A second dipole magnet sandwiched between four proportional wire chambers (P1-P4) served as spectrometer. Two gas Čerenkov counters C1 and C2, filled with CH_4 at atmospheric pressure, and an electromagnetic calorimeter distinguished $\pi^\pm$, and $\mu^\pm$ from $e^\pm$. $\pi^\pm$ are separated from $\mu^\pm$ by a set of 12 muon chambers. Four hodoscopes were added to the detector for trigger purposes. In our analysis we determined the K^+ momentum using the beam line as a spectrometer, the position of the decay vertex and the information from the pixel counter installed just upstream of the decay volume.

The first level trigger selected three charged particle tracks based on coincidences between the A-, and D-hodoscope and the calorimeter. The second level trigger indicated the presence of an e^+ not accompanied by an e^-. It required signals in both right side and only minimal light in both left side Čerenkov counters. In this we discriminated against two of the most common background channels: (1) $K^+ \to \pi^+\pi^+\pi^-$ (K_τ) and (2) $K^+ \to \pi^+\pi^0$ followed by $\pi^0 \to e^+e^-\gamma$ (K_{dal}).

The offline analysis selected events containing three charged tracks with a vertex within the decay volume of acceptable quality, a summed momentum of less than 5.87 GeV/c, and a timing spread between the tracks consistent with the resolution of 0.5 ns. Even after particle identification criteria were applied, the remaining sample still contained background events mainly from K_τ-decay with a misidentification of a π^+ as an e^+ and accidentals. Requiring that the K^+ reconstructed from the three charged daughter particles does not track back to the target reduced the background from K_τ to the level of 1.3±0.3%, since for K_{e4} the undetected neutrino made the reconstruction incomplete. The dominating accidental background was combination of a $\pi^+\pi^-$ pair from a K_τ decay with an e^+ from either the beam or a coincident decay with an e^+ in its final state. A likelihood method was employed to reduce this background to a level of 2.4±1.2%. Due to the excellent particle identification capabilities of our detector all other backgrounds were negligible.

After the event selection 406,103 events remained, of which we estimate 388270 ± 5025 to be K_{e4} events.

To determine the branching ratio, the form factors and other related quantities a Monte Carlo simulation is needed. Our code, based on GEANT, takes into account the detector geometry as well as the independently measured efficiencies of all detector elements. K_{e4} decays are modeled by ChPT on the one loop level [15,16]. Radiative corrections are included following Diamant-Berger [17]. With this apparatus, we generated $81.6 \cdot 10^6$ K_{e4} events, resulting in $2.9 \cdot 10^6$ accepted events. The agreement between data and Monte Carlo in all control variable distributions is very good, as e.g. evidenced by the plots shown in Fig. 3.

The K_{e4} branching ratio is measured with respect to K_τ decay. K_τ events were collected in a minimum bias prescaled trigger together with K_{e4} events. With $Br(\tau) = (5.59 \pm 0.05)\%$ [18], the K_{e4} branching ratio is calculated to be

$$BR(K_{e4}) = (4109 \pm 8\,(\text{stat.}) \pm 110\,(\text{syst.})) \cdot 10^{-8} .$$

This result agrees well with the average of previous experiments [18]: $(3.91 \pm 0.17) \cdot 10^{-5}$. The systematic uncertainties are dominated by the uncertainties in the Čerenkov counter efficiencies and background contributions.

The kinematics of K_{e4} decay can be fully described by five variables [19]: (1) $s_\pi = M^2_{\pi\pi}$, and (2) $s_e = M^2_{e\nu}$, the invariant mass squared of the dipion and the dilepton, respectively; (3) θ_π and (4) θ_e, the polar angles of π^+ and e^+ in the dipion and dilepton rest frames measured with respect to the flight direction of dipion and dilepton in the K^+ rest frame, respectively; (5) ϕ, the azimuthal angle between the dipion and dilepton planes. The FWHM resolution of the apparatus for these five variables is estimated to be: 0.00133 GeV^2 (s_π), 0.00361 GeV^2 (s_e), 147 mrad (θ_π), 111 mrad (θ_e), and 404 mrad (ϕ).

The matrix element in terms of the hadronic vector and axial vector current contributions V^μ and A^μ is given by

$M_{\pi\pi}$ ($\overline{M_{\pi\pi}}$)	280-294 (285.2) MeV	294-305 (299.5) MeV
F	$5.832 \pm 0.013 \pm 0.080$	$5.875 \pm 0.014 \pm 0.083$
G	$4.703 \pm 0.089 \pm 0.069$	$4.694 \pm 0.062 \pm 0.067$
H	$-3.74 \pm 0.80 \pm 0.18$	$-3.50 \pm 0.52 \pm 0.19$
δ	$-0.016 \pm 0.040 \pm 0.002$	$0.068 \pm 0.025 \pm 0.001$
χ^2/NdF	1.071	1.080
$M_{\pi\pi}$ ($\overline{M_{\pi\pi}}$)	305-317 (311.2) MeV	317-331 (324.0) MeV
F	$5.963 \pm 0.014 \pm 0.090$	$6.022 \pm 0.016 \pm 0.094$
G	$4.772 \pm 0.054 \pm 0.070$	$5.000 \pm 0.051 \pm 0.082$
H	$-3.55 \pm 0.44 \pm 0.20$	$-3.63 \pm 0.41 \pm 0.023$
δ	$0.134 \pm 0.019 \pm 0.002$	$0.160 \pm 0.017 \pm 0.002$
χ^2/NdF	1.066	1.103
$M_{\pi\pi}$ ($\overline{M_{\pi\pi}}$)	331-350 (340.4) MeV	> 350 (381.4) MeV
F	$6.145 \pm 0.017 \pm 0.096$	$6.196 \pm 0.020 \pm 0.083$
G	$5.003 \pm 0.049 \pm 0.083$	$5.105 \pm 0.050 \pm 0.074$
H	$-1.70 \pm 0.41 \pm 0.024$	$-2.23 \pm 0.48 \pm 0.033$
δ	$0.212 \pm 0.015 \pm 0.003$	$0.284 \pm 0.014 \pm 0.003$
χ^2/NdF	1.093	1.034

TABLE I. Form factors and phase shifts $\delta \equiv \delta_0^0 - \delta_1^1$ for the six bins in $M_{\pi\pi}$. The number of degrees of freedom for each fit is 4796. The first uncertainty is statistical, the second systematical with dominant contributions from background and Čerenkov efficiency.

$$M = \frac{G_F}{\sqrt{2}} V_{us}^* \overline{u}(p_\nu)\gamma_\mu(1-\gamma_5)v(p_e)(V^\mu - A^\mu) \ , \quad (1)$$

$$A^\mu = FP^\mu + GQ^\mu + RL^\mu \ , \ \ V^\mu = H\epsilon^{\mu\nu\rho\sigma}L_\nu P_\rho Q_\sigma \ , \quad (2)$$

where $P = p_1 + p_2$, $Q = p_1 - p_2$, and $L = p_e + p_\nu$, and p_1, p_2, p_e, and p_ν are the four-momenta of the π^+, π^-, e^+, and ν_e in units of M_K, respectively. The form factors F, G, R and H are dimensionless complex functions of s_π, s_e and θ_π. The expressions for the decay rate derived from this matrix element have been given in ref. [20].

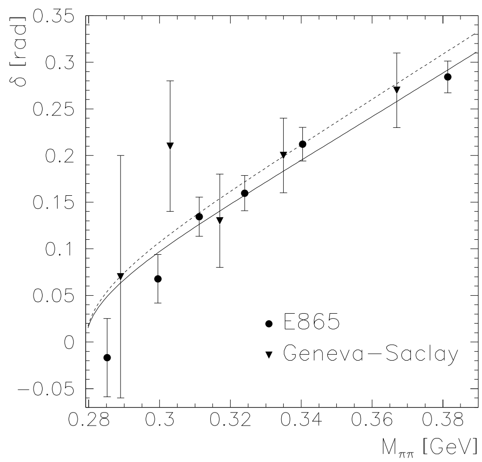

FIG. 2. Phase shift difference δ as a function of dipion mass. The dashed line represents the fit to Eq. 4 for the Geneva-Saclay data [2] and the solid line for our data as a function of the scattering length a_0^0.

Amorós and Bijnens recently developed a parameterisation of these form factors, based on a partial wave expansion in the variable θ_π [21]:

$$F = \left(f_s + f_s'\, q^2 + f_s''\, q^4 + f_e\, s_e\right) e^{i\delta_0^0}$$
$$+\tilde{f}_p\, (Q^2/s_\pi)^{1/2}\, (P\cdot L)\, \cos\theta_\pi\, e^{i\delta_1^1} \ ,$$
$$G = \left(g_p + g_p'\, q^2 + g_e\, s_e\right) e^{i\delta_1^1},\ H = \left(h_p + h_p'\, q^2\right) e^{i\delta_1^1} \ , \quad (3)$$

where $q = (s_\pi/(4M_\pi^2) - 1)^{1/2}$ is the pion momentum in $\pi\pi$ rest frame. The form factor R enters the decay distribution multiplied by m_e^2 and can therefore be neglected. This parameterisation yields ten new form factors f_s, f_s', f_s'', f_e, $\tilde{f}_p$, g_p, g_p', g_e, h_p, and h_p', which do not depend on any kinematic variables, plus the phases δ_0^0 and δ_1^1, which are functions of s_π.

The phase shifts can be related to the scattering lengths. A recent analysis [10] used the parameterisation proposed by Schenk [22]:

$$\tan\delta_\ell^I = \sqrt{1 - \frac{4M_\pi^2}{s}}\ \sum_{k=0}^{3} A_{\ell k}^I q^{2(\ell+k)} \left(\frac{4M_\pi^2 - s_\ell^I}{s - s_\ell^I}\right) \ . \quad (4)$$

The Roy equations [9] are then solved numerically, expressing the parameters $A_{\ell k}^I$ and s_ℓ^I as functions of the scattering lengths a_0^0 and a_0^2. The possible values of the scattering lengths are restricted to a band in the a_0^0 versus a_0^2 plane. The centroid of this band, the *universal curve* [23] relates a_0^0 and a_0^2:

$$a_0^2 = -0.0849 + 0.232\, a_0^0 - 0.0865\, (a_0^0)^2 \ . \quad (5)$$

For the fits we divided our data into six bins in s_π, five in s_e, ten in $\cos\theta_\pi$, six in $\cos\theta_e$ and 16 in ϕ. In the χ^2 minimisation procedure, the number of measured events in each bin j is compared to the number of expected events given by:

$$r_j = Br(K_{e4})\frac{N^K}{N^{MC}} \sum \frac{J_5(F,G,H)^{new}}{J_5(F,G,H)^{MC}} \ , \quad (6)$$

where the sum runs over all Monte Carlo events in bin j. N^K is the number of K^+ decays derived from the number of K_τ events. N^{MC} is the number of generated events. $J_5(F,G,H)^{MC}$ ($\equiv I$ [20]) is the five-dimensional phase space density generated at the momentum $q = q^{MC}$ with the form factors F, G, and H used to simulate the event. $J_5(F,G,H)^{new}$ is calculated at q^{MC} with F, G, H evaluated from the parameters of the fit. Thus, we apply the parameters on an event by event basis, and, at the same time, we divide out a possible bias caused by the matrix element, making the fit independent of the ChPT ansatz used to generate the MC.

For the fit, we have assumed that F, G, and H do not depend on s_e and that F contributes to s-waves only, i.e. $f_e = g_e = \tilde{f}_p = 0$. Our first set of fits is done independently for each bin in s_π. The above assumptions then leave one parameter each to describe F, G, and H aside from the phase difference $\delta \equiv \delta_0^0 - \delta_1^1$. The results are listed in Table I. The centroids of the bin ($\overline{M_{\pi\pi}}$) are determined following Lafferty and Wyatt [24]. If the six phase shifts in Table I are fit using Eq. 4 and Eq. 5, one obtains $a_0^0 = 0.229 \pm 0.015$ (χ^2/NdF = 4.8/5). The resulting curve is shown in Fig. 2.

f_s	$5.75 \pm 0.02 \pm 0.08$	f_s'	$1.06 \pm 0.10 \pm 0.40$
f_s''	$-0.59 \pm 0.12 \pm 0.40$	g_p	$4.66 \pm 0.05 \pm 0.07$
g_p'	$0.67 \pm 0.10 \pm 0.04$	h_p	$-2.95 \pm 0.19 \pm 0.20$
a_0^0	$0.228 \pm 0.012 \pm 0.004$		

TABLE II. Form factors and scattering length a_0^0 in the parameterisation of Eq. 3 (χ^2/NdF = 30963/28793.)

$\tilde{f}_p$	$-4.3 \pm 1.3 \pm 3.4$	$\chi^2 = 30952$
f_e	$-4.1 \pm 1.3 \pm 3.1$	$\chi^2 = 30954$
g_e	$0.5 \pm 4.4 \pm 11.3$	$\chi^2 = 30963$

TABLE III. Fit of form factors $\tilde{f}_p$, f_e, and g_e. The systematic uncertainties are dominated by the resolution of the neutrino mass squared. (NdF=28792)

We have also made a single fit to the entire data sample. In this second fit we substituted δ in Eq. 3 by the expression of Eq. 4. With the relation between a_0^0 and a_0^2 given by Eq. 5 only f_s, f_s', f_s'', g_p, g_p', h_p, and a_0^0 then remain as free parameters. The results, listed in Table II are in an excellent agreement with the ones derived in the previous paragraph.

To check the assumption $f_e = g_e = \tilde{f}_p = 0$ we also allowed these form factors to vary, one at the time, in our second fit. Table III shows that all three form factors are consistent with zero. The quality of the fits is demonstrated in Fig. 3, where the invariant mass (s_π) and azimuth (ϕ) distribution from data are compared to the reweighted Monte Carlo distributions (Eq. 6).

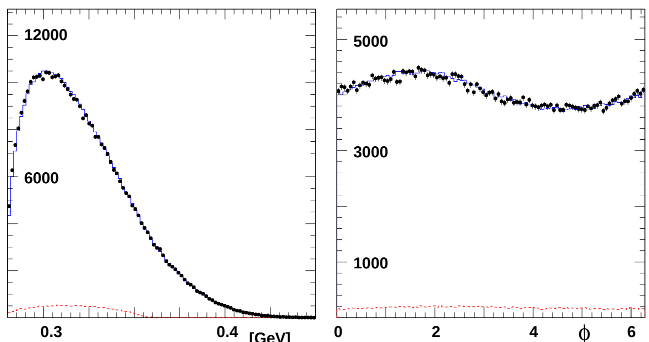

FIG. 3. Invariant mass and angle distributions describing the K_{e4} decay. The histograms are the Monte Carlo distributions while the markers with the error bars represent the data. The dashed histogram indicates the non-K_{e4} background.

The asymmetry of the ϕ distribution is the only observable, which directly depends on the phase shifts [20]. The amplitude of this asymmetry amounts to only 10% of the ϕ independent part. This explains why the statistical error of a_0^0 is still limited to 5.3%.

To summarize, experiment E865 has collected a K_{e4} event sample more than ten times larger than all previous experiments combined. From the model independent analysis of this data the momentum dependence of the form factors of the hadronic currents as well as $\pi\pi$ scattering phase shifts have been extracted. The form factors and phase shifts serve as an important input in the program to determine the couplings of the effective Hamiltonian of chiral QCD perturbation theory at low energies [25]. From a preliminary communication of these results already tight bounds on the value of the quark condensate have been extracted [13]. Using the relations between a_0^0 and a_0^2 given by the Roy equations [10], we have extracted the most precise value of the $\pi\pi$ scattering length a_0^0. This value agrees well with predictions obtained in the framework of ChPT [8].

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[1] R.W. Birge *et al.*, Phys. Rev. **139**, 1600 (1965); R.P. Ely *et al.*, Phys. Rev. **180**, 1319 (1969); W. Schweinberger *et al.*, Phys. Lett. **36B**, 246 (1971); M. Bourquin *et al.*, Phys. Lett. **36B**, 615 (1971); P. Basile *et al.*, Phys. Lett. **36B**, 619 (1971); A. Zylberstein *et al.*, Phys. Lett. **38B**, 457 (1972); E.W. Beier *et al.*, Phys. Rev. Lett. **29**, 511 (1972); ibid.**30**, 399 (1973).
[2] L. Rosselet *et al.*, Phys. Rev. **D15**, 574 (1977).
[3] E.P. Shablin, JETP **17**, 517 (1963).
[4] S. Weinberg, Physica **96A**, 327 (1979); J. Gasser, and H. Leutwyler, Ann. Phys. **158**, 142 (1984); Nucl. Phys. **B250**, 465 (1985).
[5] S. Weinberg, Phys. Rev. Lett. **17**, 616 (1966).
[6] J. Gasser, and H. Leutwyler, Phys. Lett. **B125**, 325 (1983).
[7] J. Bijnens *et al.*, Phys. Lett. **B374**, 210 (1996); Nucl. Phys. **B508**, 263 (1997); err. ibid. **B517**, 639 (1998).
[8] G. Colangelo, J. Gasser, and H. Leutwyler, Phys. Lett. **B488**, 261 (2000).
[9] S.M. Roy, Phys. Lett. **36B**, 353 (1971).
[10] B. Ananthanarayan *et al.*, Phys. Rep., in print, hep-ph/0005297.
[11] see compilation of M.M. Nagels *et al.*, Nucl. Phys. **B147**, 189 (1979); and references mentioned therein.
[12] N.H. Fuchs, H. Sazdjian and J. Stern, Phys. Lett. **B269**, 183 (1991); M. Knecht *et al.*, Nucl. Phys. **B457**, 513 (1995); ibid. **B471**, 445 (1996).
[13] G. Colangelo, J. Gasser, and H. Leutwyler, Phys. Rev Lett. **86**, 5008 (2001).
[14] R. Appel *et al.*, Nucl. Instr. Meth. **A**, in print.
[15] J. Bijnens, Nucl. Phys. **B337**, 635 (1990).
[16] C. Riggenbach *et al.*, Phys. Rev. D **43**, 127 (1991).
[17] A.M. Diamant-Berger, Ph.D. thesis, Centre d'Etudes Nucléaires de Saclay, 1976.
[18] D.E. Groom *et al.*, Eur. Phys. J. **C15**, 1 (2000).
[19] N. Cabibbo, and A. Maksymowicz, Phys. Rev. **137**, B438 (1965).
[20] A. Pais, and S.B. Treiman, Phys. Rev. **168**, 1858 (1967).
[21] G. Amorós, and J. Bijnens, J. Phys. **G25**, 1607 (1999).
[22] A. Schenk, Nucl. Phys. **B363**, 97 (1991).
[23] D. Morgan, and G. Shaw, Nucl. Phys. **B10**, 261 (1969).
[24] G.D. Lafferty, and T.R. Wyatt, Nucl. Instr. Meth. **A355**, 541 (1995).
[25] G. Amoros, J. Bijnens, and P. Talavera, Phys. Lett. **B480**,71 (2000); Nucl. Phys. **B595**, 293 (2000); err. **B598**, 665 (2001); ibid. **B602**, 87 (2001).